Bayesian Inference

- Contrasts with ‘classical’ (frequentist) statistics
- Key difference is how ‘uncertainty’ is handled

P – probability

H - hypothesis

D - data

Frequentist:

- Model parameters are fixed, but unknown
- Uncertainty is expressed as variability in hypothetical data sets: **P(D|H)**
- Make probability statements about the data, not model parameters
- Adequacy of a model tested with hypothesis testing and *P*-values; based on the repeatability of observing the data given the model

- What is a P-value?

  "if we were to repeat an experiment a large number of times, then in 5% of cases we would get a larger t-value"

- It does <u>not</u> mean: "there is a 95% probability the regression parameter lies between *x* and *y*"
- Compatibility of the data with the null hypothesis

- Fisher believed a *P* value was a <u>rough</u> guide of the strength of evidence against the null hypothesis
- *P* < 0.05 shows we should repeat the experiment - if subsequent studies also ‘significant’, unlikely to be chance
- Does <u>not</u> provide the probability of the null hypothesis

Bayesian:

- Assume model parameters are unknown (and vary), but we can estimate their distributions with data, which are fixed: **P(H|D)**
- Can make statements about model parameters with confidence
- Based on Bayes theorem – permits prior information

Baye's rule

- Obtain a 'posterior probability' based on:

1. prior probability
2. likelihood function

Likelihood

How probable is the data if the hypothesis is true?

Prior

How probable was the hypothesis before collecting data?

$$P(H|D) = \frac{P(D|H) \cdot P(H)}{P(D)}$$

Posterior

How probable is the hypothesis given the observed data?

Marginal

How probable is the new data for all possible hypotheses?

Why use Bayesian inference?

- Directly calculate the probability that an hypothesis is true
- We can have multiple well-defined hypotheses
- We can incorporate prior information (‘priors’)
- With few data, but good priors, we can draw sensible conclusions
- For some analyses there is no alternative

Disadvantages

- Priors are subjective (that is also an advantage)
- Calculations are computationally intensive (e.g. using MCMC, though INLA is not)
- Journal reviewers in fisheries biology don’t understand it! (they are learning)

Choice of priors

- Key point in Bayesian inference
- ‘Expert’ opinion
- Published studies/data
- Empirical data-derived priors
- Controversial

Advantage of informative priors

- Increase model precision
- Reduced sample size

Disadvantages?

- Potential reduced accuracy
  - (Though evidence suggests not)

Time series analysis

- Wide variety of techniques (and data types)
- A goal of time series analysis is forecasting

Time series analysis

- A set of observations collected at regular intervals
- Ideally at least 15 measurements
- Trends are forced by persistent effects (such as fishing, habitat deterioration)
- May be linear or non-linear
- Termed ‘secular’ trends

Time series analysis

- There may be seasonal and or cyclical patterns – consistently recurring
  - Seasonal are annual
  - Cyclical may be longer than annual
- There may (will) be irregular events imposing additional (unpredictable) variation

Time series analysis

- Observations close together in time tend to be correlated (serially dependent)
- An outcome is temporal dependency (pseudoreplication)
- Like a random effect, we need a temporal dependency structure in our model

Time series analysis

- Model as a ‘random walk’
- Population size in 2001 dependent on size in 2000
  - (Stock dependency in fisheries)
- Dependency decays with time
  - (population size in 2001 less dependent on size on 1990 than 2000)

Time series analysis

- Catch is modelled as a function of:
  - intercept
  - covariates
  - a trend
  - noise (mean of zero, normal distribution)

Spatial-temporal model

- Data often sampled at multiple locations through time
- Need temporal-spatial model
- Possible with Bayesian model in INLA

See:

Izquierdo *et al*. (2022). Bayesian spatio-temporal CPUE standardization: Case study of European sardine (*Sardina pilchardus*) along the western coast of Portugal. *Fisheries Management and Ecology* 29: 670-680.